**036-M-22**

**Swing up and Stabilization Inverted Pendulum Using Switching Control**

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**Abstract**

An inverted pendulum is an unstable nonlinear system widely applied to test control performance. This study proposes the effectiveness of switching control to swing up the pendulum to the upright position by using the energy controller; after that, the PD controller tries to stabilize the pendulum in the upright position with the desired arm position with the second PD controller. The numerical simulation was performed with MATLAB. The numerical results show that the simple controller will work well for exactly known system parameters or actual parameters lower than 30% of nominal value. The experimental platform was set up and some system parameters were determined. The numerical simulations base on our experimental platform parameters and sensors. Sensors include angular position sensors and angular velocity sensors for both arm and pendulum rotations.

**Introduction**

The inverted pendulum is one attractive platform in the current research because it is a nonlinear and unstable system. It is applied to test the performance of the control system. Carvalho applied a neural network to model an inverted pendulum for stabilization. Jain et al. (2021) used the fuzzy controller to swing up twin pendulums and stabilize them with on need to know the exact model. Swing up control inverted pendulum by Nguen et al. (2021).

Not only cart inverted pendulum or inverted rotary pendulum, but the wheeled inverted pendulum is also interested in the study of an inverted pendulum. Huang et al. (2021) developed two wheels to balance the inverted pendulum.

In this study, we develop the rotary inverted pendulum platform. Then apply control to swing up the pendulum close to the desired states. After that, the stabilizer controller needs to stabilize the inverted pendulum upright. The study will limit the saturated torque and see what the results of the control system will be. The control technique that moves the pendulum from downward to an upright position has many proposed methods, such as Astrom and Furuta (2000).

This study presents the inverted pendulum mathematical model, operational amplifier circuit for power amplifier control, Energy swings up inverted pendulum control, stabilized inverted pendulum control, numerical simulation for the model, experimental verification, and control performance, and conclusions.

**Mathematical Model of Swing up Inverted Pendulum**

Figure 1 shows a schematic diagram of the inverted pendulum. X0, Y0, and Z0 is a fixed coordinate system. The arm with coordinate X1, Y1, and Z1 rotates about Z0 -axis and the pendulum is rotate about Y1-axis. Coordinate X2, Y2, and Z2 is a fix on rotating arm to show how pendulum move with respect to it. Cylindrical coordinate will be applied to represent the position of use pendulum .



Y0, Y1, Z2

Z0, Z1

X0, X1

X2

Y2





Top view

Figure 1 Diagram of Inverted Pendulum.

The  position and velocity of the arm in the cylindrical coordinate, Hibbeler (2006), can be written as

 (1)

, where *L1* is the arm’s length, and ,  are the unit vector in radius and angle directions. The center of mass of the arm will be substitute the length *L1*will be *LC1*. The position and velocity of the pendulum are

 (2)

The third term of the pendulum position is in the angle direction as shown in Figure 1, where  and  are the position and velocity of the position P at the length L2 of the pendulum. If we consider the center of mass of pendulum *L2* will be substitute with *LC2*.

The kinetic energy (KE) of the arm and pendulum can be written as

 (3)

The potential energy of the arm is zero because the mass of the arm does not change in the Z1-axis or vertical axis. The potential energy of the pendulum can be written as

 (4)

, where g is gravitational acceleration = 9.81m/s2, and mp is the pendulum mass.

Dissipation energy, D, of the arm and pendulum rotation can be written as

 (5)

, where ba and vp is the viscosity friction coefficient at the motor and pendulum respectively.

To find the equation of motions of arm and pendulum, Euler-Lagrange’s method, Stutts (2017), with two degrees of freedom for arm rotation, and pendulum rotation. Euler-Lagrange Equation is

 (6)

, where i = 1, 2 . *T1* is applied torque in axis of and *T2* is zero, an applied torque in axis . Apply Equation (6) with Equation (3), (4), and (5) will get the results

 (7)

Equation (7) can be written in a matrix form

 (8)

, where



Equation (8) can be written in to state equation

 (9)

,where 

In this study, the DC servomotor will apply torque to rotate the arm. DC motor will be drived by using a power amplifier with velocity control controlled by an operational amplifier circuit, as shown in Figure 2.



Figure2 Operational amplifier circuit motor speed control of power amplifier

The combination of the operational amplifier circuit, power amplifier, and dc motor we can be written in a block diagram shown in Figure 3.

Tload



I

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KT









Figure 3 Block diagram of motor speed control with power amplifier

The block diagram has some variable values shows in Equation (10)

 (10)

, where Rs is the current sensors that flow through the motor,  is the gain of tachometer for the motor speed sensor, Kt is the motor constant, and is the desired speed.

Our objective, we need to apply control torque to the arm of the inverted pendulum. The DC servo motor is under speed control. We want to find the relationship between motor speed and the motor's generated torque. The block diagram of motor control can be written in Figure x.

 (11)

At steady state (pole lead-lag compensation >> pole of motor bm/Jm ), s = 0,

 (12)

With resistance R1, R2, R3, R4, R5, R6, R7, R8, R9, and Rs of. The capacitors C1 and C2 are 3.3 microfarads. The motor constant is 0.36 N-m/A, calculated from motor torque 50lb-ft at15.7 ampere.

The motor speed control can be written into a block diagram, as shown in Figure 4.



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Figure 4 Motor current control loop

, where



From Equation (12), the value of  is small with respect to *Vref*, then generated torque can be approximate to be 

**Swing-up Controller**

The controller will be divided into two stages. The first controller tries to bring the pendulum from angle 180 degree to be close to 0 degree. Then the second controller needs to keep the pendulum in the upright position. The energy method proposed by Astrom and Furuta (2000) will be selected as the first controller because it is easy to implement. The control signal derived by using Lyapunov stability as shown in Equation (13).

 (13)

, where *Ka* is the controller gain, is the reference energy of the pendulum shown in Equation (14), and  is the total energy of the pendulum shown in Equation (15).



(14)

The reference energy is the energy for keeping the pendulum in the upright position.

 (15)

The controller gain, *Ka,* is related to how fast that we want to make the pendulum from 180 degree to the upright position.

**Stabilization Controller**

The stabilizer controller needs to stabilize the pendulum after the swing-up controller brings the pendulum close to zero degree. This controller also needs to control the arm position. The proportional and derivative (PD) controller will be applied for pendulum stabilization and arm position control. It is easy implement. The control signal will be selected as

 (17)

, where  and  are the proportional and derivative gain for the arm position control,  and  are the proportional and derivative gain for the pendulum stabilization control, and  is the desired arm position.

**Model Verification**

A numerical simulation was performed with MATLAB. First, the correction of the mathematical model, Equation (9), was compared with the experimental result. The simulation result is shown in Figure 5 with the initial condition of angle  degree.

|  |  |
| --- | --- |
| a)Numerical simulation | b) Experimental result |

Figure 5 angular velocity of the arm when the initial position of the pendulum is 90o

**Numerical Simulations**

In this study, the experimental platform will be developed including power amplifier. The physical parameter are measured and estimated such as the motor viscosity damping as shown in Figure 6. However the system parameters may not be accurate enough then the flexible controller will be selected as shown in the previous section.

Figure 6 Friction torque and motor speed

|  |  |
| --- | --- |
| 1. State response of system of system in Equation (9) | 1. Control torque |
| c) Energy of pendulum | |

Figure 7.Time response with exactly known system parameters

The nonlinear model, Equation (9) with parameters in Table 1, will be tested with the selected controller. In the numerical simulation, swing up controller, energy control, will try to bring the pendulum from radian to upright position within angle within  radian. After that the stabilizer controller, PD control, will stabilize the pendulum and also arm position controller, PD controller, will maintain the arm position to be the desired value, in the experiment will have the desired position zero radian.

The swing pendulum controller, Equation (13), will have no control signal when the pendulum has zero angular velocity. In this simulation, the controller will move the arm motor with constant torque to have deviation from static balance by  radians. The numerical simulation will perform with exactly known system parameters as shown in Figure 7. For the actual mass of pendulum lesser than 30% of the nominal value, the result shows in Figure 8.

When mass was 30% lower than the nominal value. The controller still performance well. The swing up controller try to bring the pendulum up, then the stabilizer PD controller try to keep the pendulum in the upright position with lower energy than the nominal value shown in Figure 8 c. The simulations show that the better performance will be for the better known system parameter.

|  |  |
| --- | --- |
| 1. State response of system of system in Equation (9) | 1. Control torque [N-m] |
| c) Energy of pendulum | |

Figure 8.Time response with mass of pendulum lesser than 30% of nominal value

From the simulation, we observe that the gain of PD of pendulum stabilization and the gain of PD of arm position control on control performance. If the proportional gain of the PD of pendulum stabilization is low and low torque limitation applied to the arm, the system will not stabilize the pendulum. This is due to the large drift of the arm position to bring the pendulum in the upright position. To fix this effect by increasing the proportional gain of pendulum stabilization and/or increase the derivative gain of the arm position controller.

**Experimental Setup**

The development of swing up inverted pendulum is shown in Figure 8. System parameters are shown in Table 1.

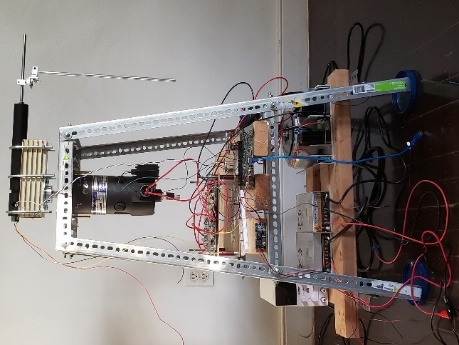


Figure 9 Experimental setup of a rotary inverted pendulum.

Table1. System parameters of the developed platform

|  |  |
| --- | --- |
| Parameters | Values |
| Motor moment of inertia, Jm | 1.83x10-3 Kg-m2 (measured from the experiment) |
| Motor viscosity damping | Ba = 0.001 (N-m)/(rad/s) |
| Pendulum viscosity damping | Bp = 0.0001 (N-m)/(rad/s) |
| Motor constant, Kt | 0.36 N-m/A |
| Arm weight | ma = 1.4kg |
| Arm moment of inertia | Ja = 0.026Kg-m2 |
| Inverted pendulum weight | mp = 0.22 Kg |
| Inverted moment of inertia | 8.6x10-3 kg-m2 |
| L1 | 0.34 m |
| L2 | 0.40 m |
| Lc1 | 0.14 m |
| Lc2 | 0.18 m |

The work done for this experimental set up is used to find the damping viscosity of motor as shown in Figure 6. Sensors include tachometer with sensitivity of 1 V/100rpm (95mV/rad/s) for motor angular velocity, and sensitivity of 2.2 V/rpm (210mv/rad/sec) for the pendulum angular velocity. The motor position and pendulum position are measured by using potentiometers. The controller will be performed by laptop and communication with the sensors and motor driver by using NI-USB6299 data acquisition.

**Conclusions**

Numerical simulations show that the energy swing-up controller and PD controller for stabilizer are work well with nonlinear mathematical model of pendulum. These controllers perform well with exactly known system parameters or deviation from the nominal value. In this study, a mathematical model of inverted pendulum was derived and verify with the experiment. The future work wants to make the experiment with the setup equipment with the proposed controller. Also, machine learning development very extensively and fast, we also plan to apply machine learning with our experimental setup.

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